OCR
Oxford Cambridge and RSA

Wednesday 6 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 The equations of two planes, P and Q, are as follows.

$$P: \quad x + 2y - 2z = 11$$
$$Q: \quad 2x - y + z = 7$$

The planes intersect in the line L.

(i) Find a cartesian equation for L. **[5]**

(ii) The point with coordinates $(a, 1, 1)$ is equidistant from P and Q. Find the two possible values of a. **[4]**

The points B and C have coordinates $(1, 2, 7)$ and $(1, 0, -5)$ respectively.

(iii) Show that B lies on Q but not P and that C lies on P but not Q. Explain why this means that the lines BC and L are skew. **[2]**

(iv) Find the shortest distance between the lines BC and L. **[5]**

The point E is the mirror image of C in the plane Q, and O is the origin.

(v) Find the volume of the tetrahedron OBCE. **[8]**

Option 2: Multi-variable calculus

2 The surface S has equation $z = x + 4x^2y - 2y^2 + 2$.

(i) Show that the tangent plane to the surface at the point $(1, 1, 5)$ has equation $z = 9x - 4$. **[5]**

(ii) Show also that if a tangent plane to S has equation $z = 9x + k$ then the only possible value for k is -4. **[3]**

(iii) A point on the surface has coordinates $(1 + a, 1 + a, 5 + b)$ where a and b are small. Show that $b \approx \lambda a$, where λ is a constant to be determined. **[3]**

(iv) Find the coordinates of the points on the surface at which the normal line is parallel to the vector $\mathbf{i} + 16\mathbf{j} - \mathbf{k}$. **[4]**

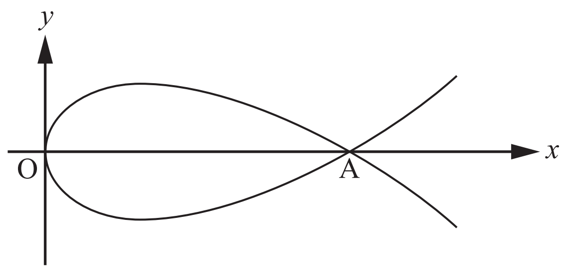
(v) Show that the only stationary point, A, on S has coordinates $\left(-\frac{1}{2}, \frac{1}{4}, \frac{13}{8}\right)$.

By finding the cross-sections through A parallel to $x = 0$ and $y = 0$ respectively, determine the nature of this stationary point. **[9]**

Option 3: Differential Geometry

3 **(a)** Prove by integration that the surface area of a sphere with radius a is given by $S = 4\pi a^2$. **[6]**

(b) A curve has parametric equations $x = 6t^2$, $y = 4t - 3t^3$. The curve crosses the x-axis at the origin O and at the point A, as shown in the diagram.



Find

(i) the values of t at the point A, **[2]**

(ii) the length of the arc OA for which t is positive, **[6]**

(iii) the radius and centre of curvature at the point where $t = \frac{1}{3}$. **[10]**

Option 4: Groups

4 You are given that the set {1, 2, 4, 7, 8, 11, 13, 14} together with the binary operation of multiplication modulo 15 forms a group G.

(i) Find the order of each element of G. **[4]**

(ii) (A) A subgroup of G has order n. Write down the possible values of n. **[2]**

(B) State all the proper cyclic subgroups of G. **[4]**

(iii) For each of the following three cases, determine whether the set together with the binary operation forms a group. If the set does form a group, state whether or not it is isomorphic to G, justifying your answer. (You may assume that each of the binary operations is associative.)

(A) The set {0, 1, 2, 3, 4, 5, 6, 7} together with the binary operation of addition modulo 8.

(B) The set {1, 2, 3, 4, 5, 6, 7, 8} together with the binary operation of multiplication modulo 9.

(C) The set of matrices

$$\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\right\}$$

together with the binary operation of matrix multiplication. (You may assume that the set is closed under matrix multiplication.) **[14]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 At a factory there are four security lights, A, B, C and D, only one of which is on at any time; which light is on at any time can be randomised by control equipment. A number of programs are devised so that the lights switch from one to another every minute with certain probabilities. For example, if A is on then at the next minute one of B, C or D will come on, with probabilities determined by the particular program being used.

The time after the start of a program is denoted by t minutes. Light A comes on when $t = 0$.

For program 1 the transition matrix is as follows.

$$\mathbf{P}_1 = \begin{pmatrix} 0 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0 & 0.6 & 0.4 \\ 0.3 & 0.4 & 0 & 0.2 \\ 0.5 & 0.4 & 0.3 & 0 \end{pmatrix}$$

The four rows and columns correspond to lights A, B, C, D in that order.

(i) Interpret the values in the leading diagonal, stating the run length for each light. **[2]**

(ii) Find the probabilities that A comes on at $t = 1$, 2, 3 and 4. **[4]**

(iii) The equilibrium probability for A is a. From your working in part **(ii)**, write down a range within which a lies. **[2]**

(iv) Find the probability that the light that comes on at $t = 5$ is different from the light that comes on at $t = 1$. **[5]**

For program 2 the following rules apply.

- The light following A is always B.
- The light following B is never D and is equally likely to be A or C.
- The light following C is never A and is equally likely to be B or D.
- The light following D is always C.

(v) Write down the transition matrix, $\mathbf{P}_2$, for program 2. **[2]**

(vi) For program 2 identify any absorbing states and reflecting barriers. **[3]**

(vii) Find the proportions of times that each light is on over a long period. Give your answers as exact fractions. **[6]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

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1 (i)

1 (ii)

1 (iii)

1 (iv)

1 (v)

2 (i)	
2 (ii)	

2 (iii)

2 (iv)

2 (v)

3 (a)

3 (b)(i)	
3 (b)(ii)	

3 (b)(iii)	

4 (i)

4 (ii)(*A*)	
4 (ii)(*B*)	

4 (iii)(*A*)	
4 (iii)(*B*)	

4 (iii)(*C*)	

5 (i)	
5 (ii)	

5 (iii)	
5 (iv)	

5 (v)	
5 (vi)	
5 (vii)	

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2018

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Mark Scheme

1	(i)		Find a common point. e.g. set $z=0$ then solve $x+2y=11,\ 2x-y=7$ simultaneously. $\Rightarrow (5, 3, 0)$	M1 A1	Find a common point
			Find the vector product $\Rightarrow n_1=\begin{pmatrix}1\\2\\-2\end{pmatrix},\quad n_2=\begin{pmatrix}2\\-1\\1\end{pmatrix}$	M1	Evaluation of vector product **or** finding a second point and using it to find direction of *L*
			$\Rightarrow n_1\times n_2=\begin{pmatrix}0\\-5\\-5\end{pmatrix}\equiv\begin{pmatrix}0\\1\\1\end{pmatrix}$ *Alternative scheme* **M1** Eliminating one variable **A1** For obtaining one of $x=5,\ y-z=3$ **M1** Eliminating another, or substituting **A1** For obtaining the other equation **A1** For the answer, as before	A1	
			$\Rightarrow \frac{x-5}{0}=\frac{y-3}{1}=\frac{z}{1}$ **oe** e.g. $x=5,\ y=z+3$	A1	*Do not accept vector or parametric form*
				[5]	
	(ii)		$d_1=\left\lvert\frac{2a-1+1-7}{\sqrt{1^2+2^2+1^2}}\right\rvert=\left\lvert\frac{2a-7}{\sqrt{6}}\right\rvert$	M1	Using formula for distance *(one sufficient; modulus not required;* *M0 if neither 11 nor 7 used)*
			$d_2=\left\lvert\frac{a+2-2-11}{\sqrt{1^2+2^2+2^2}}\right\rvert=\left\lvert\frac{a-11}{\sqrt{9}}\right\rvert=\left\lvert\frac{a-11}{3}\right\rvert$	A1	Both distances correct *(modulus not required)*
			$\Rightarrow \frac{2a-7}{\sqrt{6}}=\frac{a-11}{3}\Rightarrow a=\frac{21-11\sqrt{6}}{6-\sqrt{6}}=2-\frac{3}{2}\sqrt{6}$	M1	Finding **two** equations (or squaring)
			Or $\frac{2a-7}{\sqrt{6}}=\frac{11-a}{3}\Rightarrow a=2+\frac{3}{2}\sqrt{6}$	A1	Accept (art) $-1.67,\ 5.67$ *or* $\frac{8\pm\sqrt{216}}{4}$ *etc*
				[4]	

	(iii)		$1+4-14\neq 11$ but $2-2+7=7$ $1+0+10=11$ but $2-0-5\neq 7$	B1	www Needs evidence of substitution
			BC intersects *Q* at B only; and *P* at C only. B and C are not on *L* so BC does not intersect *L* BC is not parallel to *Q* (or *P*), so BC is not parallel to *L*	B1	Convincingly explain why BC does not intersect *L* **and** is not parallel to *L*
				[2]	
	(iv)		$\mathbf{d}=\text{CB}=\begin{pmatrix}1\\2\\7\end{pmatrix}-\begin{pmatrix}1\\0\\-5\end{pmatrix}=\begin{pmatrix}0\\2\\12\end{pmatrix}$, $\mathbf{e}=\begin{pmatrix}0\\1\\1\end{pmatrix}$	B1	For $\begin{pmatrix}0\\2\\12\end{pmatrix}$
			$\mathbf{d}\times\mathbf{e}=\begin{pmatrix}0\\2\\12\end{pmatrix}\times\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}-10\\0\\0\end{pmatrix}$, $\lvert\mathbf{d}\times\mathbf{e}\rvert=10$	M1 A1 ft	Vector product of directions Evaluation of vector product
			$\mathbf{a}-\mathbf{b}=\begin{pmatrix}4\\1\\-7\end{pmatrix}\Rightarrow(\mathbf{a}-\mathbf{b}).(\mathbf{d}\times\mathbf{e})$	M1	Using correct formula to evaluate the distance
			$\left\lvert\dfrac{(\mathbf{a}-\mathbf{b}).(\mathbf{d}\times\mathbf{e})}{\lvert\mathbf{d}\times\mathbf{e}\rvert}\right\rvert=4$	A1	cao
				[5]	

	(v)		$\mathbf{n}=\begin{pmatrix}2\\-1\\1\end{pmatrix}\Rightarrow$ CE has equation (CE is perpendicular to *Q*)	M1	*Alternative scheme* **M1** Finding distance from C to *Q* **A1** Distance is $\frac{5}{3}\sqrt{6}$ **A1** Length of CE is $\frac{10}{3}\sqrt{6}$ **A1** $\text{CE}=(\pm)\frac{10}{3}\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **or** E correct
			$\mathbf{r}=\begin{pmatrix}1\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}1+2\lambda\\-\lambda\\-5+\lambda\end{pmatrix}$ (Equation of CE)	A1	
			Meets *Q* when $2+4\lambda+\lambda-5+\lambda=7$ (Equation for *λ*) $\Rightarrow 6\lambda=10\Rightarrow\lambda=\frac{5}{3}$	A1	
			E corresponds to $\lambda=\frac{10}{3}$ So E has coordinates $\left(\frac{23}{3},-\frac{10}{3},-\frac{5}{3}\right)$	A1	
			Volume of tetrahedron $=\frac{1}{6}\lvert OE.(OB\times OC)\rvert$	M1	Appropriate scalar triple product stated (any 3 non-coplanar edges) (1/6 not required)
			$=\frac{1}{6}\left\lvert\left(\begin{pmatrix}\frac{23}{3}\\\frac{-10}{3}\\\frac{-5}{3}\end{pmatrix}.\begin{pmatrix}1\\2\\7\end{pmatrix}\times\begin{pmatrix}1\\0\\-5\end{pmatrix}\right)\right\rvert=\frac{1}{6}\left\lvert\left(\begin{pmatrix}\frac{23}{3}\\\frac{-10}{3}\\\frac{-5}{3}\end{pmatrix}.\begin{pmatrix}-10\\12\\-2\end{pmatrix}\right)\right\rvert$	A1 ft	Correct vectors substituted into correct formula (including 1/6)
				A1 ft	Evaluation of vector product
			$=\frac{1}{6}\left\lvert\left(-\frac{230}{3}-\frac{120}{3}+\frac{10}{3}\right)\right\rvert=\frac{340}{18}$ $\Rightarrow$ Volume of solid $=\frac{170}{9}$	A1	cao Accept 18.9
				[8]	

2	(i)		$\frac{\partial z}{\partial x}=1+8xy,\quad \frac{\partial z}{\partial y}=4x^2-4y$	B1B1	
			At P (1, 1, 5), $\frac{\partial z}{\partial x}=9,\ \frac{\partial z}{\partial y}=0$	B1	
			Tangent plane at is $z-c=\left.\frac{\partial z}{\partial x}\right\|_{(a,b)}(x-a)+\left.\frac{\partial z}{\partial y}\right\|_{(a,b)}(y-b)$	M1	**or** $z=9x+d$ **or** $9x-z=d$
			$\Rightarrow z-5=9(x-1)+0(x-1)\Rightarrow z-5=9(x-1)\Rightarrow z=9x-4$	A1	**AG**
				[5]	
	(ii)		For tangent plane to be $z=9x+k$, $\frac{\partial z}{\partial x}=9$ and $\frac{\partial z}{\partial y}=0$	M1	
			$\frac{\partial z}{\partial y}=4x^2-4y=0\Rightarrow y=x^2$,		
			$\frac{\partial z}{\partial x}=1+8xy\Rightarrow 1+8x^3=9\Rightarrow x=1$	A1	**or** $y=1$
			$\Rightarrow y=1\ \Rightarrow z=5$ So z = 9x + k only at (1, 1, 5) ⇒k = −4	A1	**AG** Must mention −4
				[3]	
	(iii)		$\delta z\approx\left.\frac{\partial z}{\partial x}\right\|_P\delta x+\left.\frac{\partial z}{\partial y}\right\|_P\delta y$ ($\delta z\approx 9\delta x+0\delta y$) $\Rightarrow b=9a+0\Rightarrow b=9a$	M1 A1 ft A1	*Alternative scheme* **M1** Substituting $x=y=1+a,\ z=5+b$ **A1** For $b=9a+10a^2+4a^3$ **A1** For $b\approx 9a$ since a is small
				[3]	

	(iv)		$\frac{\partial z}{\partial x}=1$ and $\frac{\partial z}{\partial y}=16$ $\Rightarrow 1+8xy=1 \Rightarrow xy=0 \Rightarrow x=0$ or $y=0$ $\Rightarrow 4x^2-4y=16 \Rightarrow x^2=y+4$ $x=0 \Rightarrow y=-4, z=-30$ i.e. $(0,-4,-30)$ $y=0 \Rightarrow x=\pm 2, z=0,4$ i.e. $(2,0,4)$ and $(-2,0,0)$	**M1** **A1** **A1A1**	Allow M1 for −1, −16 M0 for *λ*, 16*λ* (unless $\lambda=\pm 1$ seen later) **SC** If M1A0 then SC A1 for all *x* and *y* values correct
				[4]	
	(v)		$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0 \Rightarrow 1+8xy=0,\quad 4x^2-4y=0$ $\Rightarrow y=x^2, 1+8xy=0 \Rightarrow x^3=-\frac{1}{8} \Rightarrow x=-\frac{1}{2}$ $\Rightarrow y=\frac{1}{4}, z=-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+2=\frac{13}{8}$ $x=-\frac{1}{2}$ is the only root of the equation $1+8x^3=0$ So only one stationary point. Set $x=-\frac{1}{2} \Rightarrow z=\frac{3}{2}+y-2y^2$ Set $y=\frac{1}{4} \Rightarrow z=x+x^2+\frac{15}{8}$ One section has a maximum and the other has a minimum meaning that the stationary point is a saddle point	**M1** **M1** **A1** **A1** **A1** **M1** **A1** **A1** **A1**	For $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0$ Obtaining an equation in *x* (or y) only AG For $x=-\frac{1}{2}$ (*or* $y=\frac{1}{4}$) AG For other two coordinates *(some evidence required for z)* AG *Must refer to this being the only solution (or the only stationary point)* Finding cross-sections *(one sufficient)* *(M0 for* $x=0$ *and* $y=0$ *used)* *Correct diagrams sufficient*
				[9]	

Question			Answer	Marks	Guidance
3	(a)		Formula for surface area = $\int_{x_1}^{x_2} 2\pi y \, \mathrm{d}s$	M1	Used with equation of a circle
			$\mathrm{d}s = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ *or* $\mathrm{d}s = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$	M1	Used
			$x^2+y^2=a^2 \Rightarrow y=\sqrt{a^2-x^2}$, *or* $x=a\cos\theta,\ y=a\sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x}{y}$ *or* $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-a\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta}=a\cos\theta$	B1	(in any form)
			$S=2\int_0^a 2\pi y\sqrt{1+\frac{x^2}{y^2}}\mathrm{d}x$ *or* $S=\int_0^\pi 2\pi a\sin\theta\sqrt{a^2\sin^2\theta+a^2\cos^2\theta}\,\mathrm{d}\theta$	A1	Correct integral expression for S including limits
			$=4\pi a\int_0^a \mathrm{d}x$ $\qquad$ $=2\pi a^2\int_0^\pi \sin\theta\,\mathrm{d}\theta$	A1	For $y\,\mathrm{d}s=a\,\mathrm{d}x$ or $y\,\mathrm{d}s=a^2\sin\theta\,\mathrm{d}\theta$
			$=4\pi a^2$	A1	AG
				[6]	
	(b)	(i)	$y=0\Rightarrow 3t^3=4t$	M1	
			$t\neq 0\Rightarrow t=\pm\frac{2}{3}\sqrt{3}$	A1	Accept $\pm\frac{2}{\sqrt{3}}$ (A0 if $t=0$ included)
				[2]	
		(ii)	$x=6t^2\Rightarrow \dot{x}=12t$	B1	
			$y=4t-3t^3\Rightarrow \dot{y}=4-9t^2$	B1	
			$\dot{x}^2+\dot{y}^2=\left(4+9t^2\right)^2$	M1 A1	Substituting into $\dot{x}^2+\dot{y}^2$
			$\Rightarrow s=\int_0^{\frac{2}{3}\sqrt{3}}\left(4+9t^2\right)\mathrm{d}t=\left[4t+3t^3\right]_0^{\frac{2}{3}\sqrt{3}}=\frac{2}{3}\sqrt{3}\left(4+4\right)=\frac{16}{3}\sqrt{3}$	M1 A1	For $\int\sqrt{\dot{x}^2+\dot{y}^2}\,\mathrm{d}t$ (limits not required)
				[6]	

		(iii)	$\ddot{x}=12,\quad \ddot{y}=-18t\ (=-6)$	B1	
			$\rho=\dfrac{\left(\dot{x}^2+\dot{y}^2\right)^{3/2}}{\dot{x}\ddot{y}-\ddot{x}\dot{y}}=\dfrac{\left(4+9t^2\right)^3}{-216t^2-12\left(4-9t^2\right)}=\dfrac{\left(4+9t^2\right)^3}{-48-108t^2}$	M1 A1	Using formula for ρ or κ Correct substitution (algebraic or numerical)
			$=\dfrac{\left(4+9t^2\right)^3}{-12\left(4+9t^2\right)}=-\dfrac{1}{12}\left(4+9t^2\right)^2$		
			When $t=\dfrac{1}{3}$ $\rho=(-)\dfrac{25}{12}$	A1	
			Centre is $\mathbf{c}=\mathbf{r}+\rho\hat{\mathbf{n}}$	M1	Used
			When $t=\dfrac{1}{3},\mathbf{r}=\begin{pmatrix}\frac{2}{3}\\ \frac{11}{9}\end{pmatrix}$	B1	
			$\tan\psi=-\dfrac{4-9t^2}{12t}=\dfrac{3}{4}$	M1	Finding $\dfrac{dy}{dx}$
			$\Rightarrow \hat{\mathbf{n}}=(\pm)\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$	A1	Correct unit normal
			$\Rightarrow \mathbf{c}=\begin{pmatrix}\frac{2}{3}\\ \frac{11}{9}\end{pmatrix}-\dfrac{25}{12}\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}=\begin{pmatrix}\frac{23}{12}\\ -\frac{4}{9}\end{pmatrix}$	A1 A1	
				[10]	

4	**(i)**		Order 2: 4,11, 14 Order 4: 2, 7, 8, 13 1 | 2 | 4 | 7 | 8 | 11 | 13 | 14 1 | 4 | 2 | 4 | 4 | 2 | 4 | 2	**B4**	B3 one error, B2 2 errors, B1 3 errors
				[4]	
	(ii)	**(*A*)**	1, 2, 4, 8	**B2**	B1 for one omission
				[2]	
		(*B*)	Order 2: {1, 4}, {1, 11},{1, 14} Order 4: {1, 2, 4, 8},{1, 4, 7, 13}	**B4**	B1 one correct of order 2 B1 one correct of order 4 B1 any 4 correct B1 all 5 correct with no extras *Ignore {1} if included*
				[4]	
	(iii)		(A) Yes. Closed 0 is the identity element Each element has an inverse 1,7 2,6 and 3,5 are inverse pairs, 4 is self inverse Not isomorphic e.g. G has 4 self inverse elements, this has 2 (B) No e.g. $3 \times 3 = 0$ so set not closed under this operation	 **B1** **B1** **B1** **B1** **B1** **B1** **B1** **B1**	 Giving inverses correctly (0 not needed) With reason Reason

			(C) Yes $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is the identity All have inverses $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ is the inverse of $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ All other elements are self inverse. Not isomorphic as this one has 6 self-inverse elements	**B1** **B1** **B1** **B1** **B1** **B1**	(or vice-versa) With reason
				[14]	

5	(i)		Lights always change – they are never on for consecutive minutes. The run length is therefore 0 for each	B1 B1	 Accept run length 0 or 1
				[2]	
	(ii)		Finding powers of P_1 0, 0.27, 0.162, 0.2091	M1 A1 A1A1	 *If M1A0 then SC A1 for three of 0, 0.27, 0.162, 0.2091 regardless of times given*
				[4]	
	(iii)		 0.162 < *a* < 0.2091	M1 A1 ft	For lower limit 0 or 0.162 and upper limit 0.27 or 0.2091
				[2]	
	(iv)		$\mathbf{P}_1^4 = \begin{pmatrix} \mathbf{0.2091} & 0.1976 & 0.201 & 0.1762 \\ 0.329 & \mathbf{0.3332} & 0.257 & 0.2492 \\ 0.2088 & 0.2054 & \mathbf{0.2529} & 0.256 \\ 0.2531 & 0.2638 & 0.2891 & \mathbf{0.3186} \end{pmatrix}$ probabilities for $t = 1$ are 0, 0.2, 0.3, 0.5 $\text{P(same at } t = 1 \text{ and } 5) = 0.2 \times 0.3332 + 0.3 \times 0.2529 + 0.5 \times 0.3186 = 0.302$ $\Rightarrow \text{P(different)} = 0.698$	M1 A1 M1 M1 A1	Using **diagonal** elements from P^4. Diagonal elements correct Using probabilities from 2nd minute Method for P(same) or P(different)
				[5]	
	(v)		$P_2 = \begin{matrix} & A & B & C & D \\ A & 0 & 0.5 & 0 & 0 \\ B & 1 & 0 & 0.5 & 0 \\ C & 0 & 0.5 & 0 & 1 \\ D & 0 & 0 & 0.5 & 0 \end{matrix}$	B2	B1 one error
				[2]	
	(vi)		**(A)** There are no absorbing states. **(B)** A and D are reflecting barriers	B1 B1B1	 Maximum B1 if any extras
				[3]	

	(vii)		Algebraically: $\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}$ *Alternative scheme* **M1** Finding large even **and** large odd powers **M1** Averaging over even and odd powers **A4** Answers, which must be given as fractions	**M1** **M1** **A4**	Using column vector Creating and solving equations *(needs to use* $a+b+c+d=1$*)* A1 for each
				[6]	

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AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4757/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4757 series overview

This was the final assessment series for the unitised 3895,3898/7895,7898 GCE Mathematics (MEI) specification. There will be a resit opportunity in the summer 2019.

Further Applications of Advanced Mathematics (FP3) 4757 is an A2 GCE unit taken as part of the Mathematics (MEI) specification, occasionally used as the second optional unit in AS Further Mathematics (3896), but generally as one of the four optional units in A level Further Mathematics (7896). A few candidates are also studying AS Further Mathematics (Additional) (3897) or A level Further Mathematics (Additional) (7897), and in this case the grading optimisation process will determine which qualification this unit will contribute towards.

Further Applications of Advanced Mathematics (FP3) 4757 is made up of five optional questions from which candidates choose three. The work on this paper was generally of a high standard, with most candidates producing good answers to all of their chosen three questions. Presentation was variable, but most candidates did set out their solutions clearly.

Several questions asked for a proof or for a given result to be obtained. When answering such questions, candidates are advised to show every detail of their working. The examiner should not be expected to fill in gaps in calculations or reasoning, however 'obvious' they might seem. No credit can be given for just copying a given result, without explaining how it has arisen.

Option overview

Q2 and Q4 were the most popular two questions (77% and 74% of the candidature respectively), Question 3 was the least frequently chosen question (with only 17% of the candidates attempting this question).

There was very little difference between the average score achieved on each of the five questions, although Q3 and Q4 were answered slightly better than the other questions.

There were a small percentage of candidates that answered more than the required three questions: in these cases all questions were marked and the best three scores used. Generally, candidates are better advised to focus all their time on three specific questions rather than attempting more questions in a time designed for three full answers; any time at the end would be better used to refine their earlier answers rather than attempting an extra question.

Question 1 (i)

1 The equations of two planes, P and Q, are as follows.

$$\begin{aligned} P: &\quad x + 2y - 2z = 11 \\ Q: &\quad 2x - y + z = 7 \end{aligned}$$

The planes intersect in the line L.

(i) Find a cartesian equation for L. **[5]**

Most candidates found the direction of the line of intersection as the vector product of the two normal vectors, then found a point on the line. Others began by eliminating a variable and worked algebraically. A very large number gave the equation of the line in vector or parametric form instead of the cartesian form requested. The correct answer was $x = 5$, $y = z + 3$ seen together, although answers written in the form $\frac{x-5}{0} = \frac{y}{1} = \frac{z+3}{1}$ were condoned.

Question 1 (ii)

(ii) The point with coordinates $(a, 1, 1)$ is equidistant from P and Q. Find the two possible values of a. **[4]**

The only successful approach here was to find the distances of the point from the two planes, and then equate them. Most candidates used the standard formula and obtained a correct linear equation for *a*. To find the second value of *a* it was necessary to appreciate that the formula gives the distance as the modulus of an expression; this leads to a second linear equation, or, by squaring, to a quadratic equation. Many candidates gave the second value as minus the value already obtained.

Question 1 (iii)

The points B and C have coordinates (1, 2, 7) and (1, 0, −5) respectively.

(iii) Show that B lies on Q but not P and that C lies on P but not Q. Explain why this means that the lines BC and L are skew. **[2]**

For the first mark, candidates were required to show four substitutions. This was usually done correctly, although there were many careless arithmetic errors. For the second mark, candidates were expected to explain convincingly why BC and *L* do not intersect and are not parallel. Few candidates considered both of these requirements.

Question 1 (iv)

(iv) Find the shortest distance between the lines BC and L. **[5]**

The method for finding the shortest distance between skew lines was well understood and it was usually applied accurately.

Question 1 (v)

The point E is the mirror image of C in the plane Q, and O is the origin.

(v) Find the volume of the tetrahedron OBCE. **[8]**

The first step was to find the point E. Some candidates considered the equation of the line from C perpendicular to *Q*, finding where it meets *Q* and then going the same distance on the other side. Others used the formula to find the perpendicular distance from C to *Q*, then CE is a vector normal to *Q* with length twice this distance. Many candidates did not know how to find E, and several just wrote down that E is $(-1, 0, 5)$, which is the reflection of C in the origin O. Most candidates gave a correct scalar triple product for calculating the volume of the tetrahedron, and were able to evaluate it using the coordinates of their point E.

Question 2 (i)

2 The surface S has equation $z = x + 4x^2y - 2y^2 + 2$.

(i) Show that the tangent plane to the surface at the point (1, 1, 5) has equation $z = 9x - 4$. **[5]**

Almost all candidates found the partial derivatives correctly and evaluated them at the given point. Most candidates then obtained the equation of the tangent plane correctly.

Question 2 (ii)

(ii) Show also that if a tangent plane to S has equation $z = 9x + k$ then the only possible value for k is −4. **[3]**

Most candidates realised that the partial derivatives needed to take the same values as in part (i). It could then be shown that this occurs only at the point given in part (i) and hence *k* must be −4. Some candidates did not mention *k* in their answer, even though this was the aim of the question.

Question 2 (iii)

(iii) A point on the surface has coordinates $(1 + a, 1 + a, 5 + b)$ where a and b are small. Show that $b \approx \lambda a$, where λ is a constant to be determined. **[3]**

This was very well answered. Most candidates used the values of the partial derivatives, some substituted the coordinates into the equation of the tangent plane, and others substituted the coordinates into the equation of the surface.

Question 2 (iv)

(iv) Find the coordinates of the points on the surface at which the normal line is parallel to the vector $\mathbf{i} + 16\mathbf{j} - \mathbf{k}$. **[4]**

Most candidates knew that this required $\frac{dz}{dx} = 1$ and $\frac{dz}{dy} = 16$, and used these to find the coordinates of the three points. Some started with $\frac{dz}{dx} = -1$ and $\frac{dz}{dy} = -16$; and others with just $\frac{dz}{dx} = \lambda$ and $\frac{dz}{dy} = 16\lambda$.

Question 2 (v)

(v) Show that the only stationary point, A, on S has coordinates $\left(-\frac{1}{2}, \frac{1}{4}, \frac{13}{8}\right)$.

By finding the cross-sections through A parallel to $x = 0$ and $y = 0$ respectively, determine the nature of this stationary point. **[9]**

Most candidates were able to use $\frac{dz}{dx} = \frac{dz}{dy} = 0$ to obtain the stationary point. As this point is given on the question paper, candidates should show all their working, including the calculation of the *z*-coordinate. Very many candidates did not explain why it is the only stationary point. There were 4 marks for finding the cross-sections and determining the nature of the stationary point, and many candidates did not score any of these marks, usually by considering the sections given by $x = 0$ and $y = 0$ (which do not contain the stationary point) or by using methods which do not involve cross-sections. Candidates who considered the correct sections $x = -\frac{1}{2}$ $x = -\frac{1}{2}$ and $y = \frac{1}{4}$ were very often successful in showing that the stationary point is a saddle point.

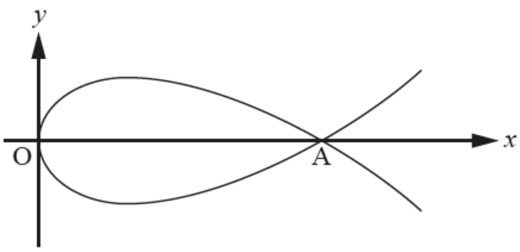
Question 3 (a)

3 **(a)** Prove by integration that the surface area of a sphere with radius a is given by $S = 4\pi a^2$. **[6]**

Most candidates attempted to find the surface area of revolution of a circle, taken as $x = a\cos\theta$, $y = a\sin\theta$ slightly more often than $y^2 = a^2 - x^2$ Most candidates successfully manipulated $\int 2\pi y \, ds$ into an integrable form, and often completed the proof correctly. A significant number of candidates did not choose appropriate limits of integration and so were unable to progress to a fully correct solution, with a small minority scoring zero.

Question 3 (b) (i)

(b) A curve has parametric equations $x = 6t^2$, $y = 4t - 3t^3$. The curve crosses the x-axis at the origin O and at the point A, as shown in the diagram.



Find

(i) the values of t at the point A, **[2]**

This was well answered. Just a few candidates included the incorrect value $t = 0$

Question 3 (b) (ii)

(ii) the length of the arc OA for which t is positive, **[6]**

Most candidates understood how to find the arc length, and carried out the process accurately.

Question 3 (b) (iii)

(iii) the radius and centre of curvature at the point where $t = \frac{1}{3}$. **[10]**

There were several steps to be completed here: finding the second derivatives and hence the radius of curvature; finding the coordinates of the point on the curve; finding the unit normal vector, and hence the coordinates of the centre of curvature. Most candidates applied the relevant techniques confidently, and there were very many completely correct solutions.

Question 4 (i)

4 You are given that the set {1, 2, 4, 7, 8, 11, 13, 14} together with the binary operation of multiplication modulo 15 forms a group G.

(i) Find the order of each element of G. **[4]**

Almost every candidate gave the orders of all the elements correctly.

Question 4 (ii) (*A*)

(ii) (*A*) A subgroup of G has order n. Write down the possible values of n. **[2]**

Most candidates gave all the factors of 8 correctly. Some discounted $n = 1$ or $n = 8$ or both, presumably because these correspond to trivial or improper subgroups, although the question did not ask for these to be excluded.

Question 4 (ii) (*B*)

(*B*) State all the proper cyclic subgroups of G. **[4]**

In this part the question does ask for only proper cyclic subgroups. Most candidates gave the three subgroups of order 2, and the two cyclic subgroups of order 4, correctly. Many also gave the non-cyclic subgroup {1, 4, 11, 14}.

Question 4 (iii)

(iii) For each of the following three cases, determine whether the set together with the binary operation forms a group. If the set does form a group, state whether or not it is isomorphic to G, justifying your answer. (You may assume that each of the binary operations is associative.)

(*A*) The set {0, 1, 2, 3, 4, 5, 6, 7} together with the binary operation of addition modulo 8.

(*B*) The set {1, 2, 3, 4, 5, 6, 7, 8} together with the binary operation of multiplication modulo 9.

(*C*) The set of matrices

$$\left\{\begin{pmatrix}1&0\\0&1\end{pmatrix}, \begin{pmatrix}0&-1\\1&0\end{pmatrix}, \begin{pmatrix}-1&0\\0&-1\end{pmatrix}, \begin{pmatrix}0&1\\-1&0\end{pmatrix}, \begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-1\\-1&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}-1&0\\0&1\end{pmatrix}\right\}$$

together with the binary operation of matrix multiplication. (You may assume that the set is closed under matrix multiplication.) **[14]**

Most candidates understood the properties which they should investigate, and presented their work clearly. Many lost marks by omitting details (for example, asserting that every element has an inverse without listing the inverses or explaining why) and by not stating their conclusion (whether or not the set is a group). For (*A*) and (*C*), which are groups, candidates needed to consider whether the group is isomorphic to *G*, and some did not attempt this. Most based their arguments on the orders of the elements, for example (*A*) has one element of order 2, (*C*) has five and *G* has three. There was some confusion between self-inverse elements (which include the identity) and elements of order 2 (which do not include the identity), but this was not penalised if the candidate's intention was clear. Other arguments used were whether or not the group is cyclic (for (*A*)), or commutative (for (*C*)). For (*B*), which is not a group, most candidates showed that the set is not closed. Others observed that, for example, the element 3 does not have an inverse. Some considered the orders of the elements (for example, 2 has order 6, which is not a factor of 8).

Question 5 (i)

5 At a factory there are four security lights, A, B, C and D, only one of which is on at any time; which light is on at any time can be randomised by control equipment. A number of programs are devised so that the lights switch from one to another every minute with certain probabilities. For example, if A is on then at the next minute one of B, C or D will come on, with probabilities determined by the particular program being used.

The time after the start of a program is denoted by t minutes. Light A comes on when $t = 0$.

For program 1 the transition matrix is as follows.

$$\mathbf{P}_1 = \begin{pmatrix} 0 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0 & 0.6 & 0.4 \\ 0.3 & 0.4 & 0 & 0.2 \\ 0.5 & 0.4 & 0.3 & 0 \end{pmatrix}$$

The four rows and columns correspond to lights A, B, C, D in that order.

(i) Interpret the values in the leading diagonal, stating the run length for each light. **[2]**

Most candidates understood that the zeros in the leading diagonal indicated that no light would stay on for two consecutive minutes, and went on to state the run lengths. Either 0 or 1 was acceptable as the value for the run lengths.

Question 5 (ii)

(ii) Find the probabilities that A comes on at $t = 1, 2, 3$ and 4. **[4]**

Most candidates calculated the four probabilities correctly.

Question 5 (iii)

(iii) The equilibrium probability for A is a. From your working in part **(ii)**, write down a range within which a lies. **[2]**

The probabilities in part (ii) are oscillating below and above the limiting value, and many candidates correctly deduced that a lies between the probabilities at $t = 3$ and $t = 4$ Some gave less efficient limits such as the probabilities at $t = 0$ and $t = 1$.

Question 5 (iv)

(iv) Find the probability that the light that comes on at $t = 5$ is different from the light that comes on at $t = 1$. **[5]**

A common error here was to find the probabilities at $t = 5$ and at $t = 1$, and then proceed as if these were independent; this approach earned just 1 mark out of 5. Those who considered the correct conditional probabilities (which are the diagonal elements of P_1^4) usually knew how to apply them, and there were a very good number of fully correct solutions.

Question 5 (v)

For program 2 the following rules apply.

- The light following A is always B.
- The light following B is never D and is equally likely to be A or C.
- The light following C is never A and is equally likely to be B or D.
- The light following D is always C.

(v) Write down the transition matrix, $\mathbf{P}_2$, for program 2. **[2]**

Almost all candidates wrote down the correct transition matrix.

Question 5 (vi)

(vi) For program 2 identify any absorbing states and reflecting barriers. **[3]**

The definitions of absorbing states and reflecting barriers were generally well understood.

Question 5 (vii)

(vii) Find the proportions of times that each light is on over a long period. Give your answers as exact fractions. **[6]**

Candidates who used an invariant column vector (and the sum of the probabilities being one) usually set up and solved the equations accurately. Some candidates who considered instead a high power of P_2 were unable to progress when they discovered that this was not of the form they were expecting. The limits for large odd and even powers are different, and candidates who realised this, and averaged the two limiting matrices, were usually able to obtain the correct limiting probabilities.

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AS & Advanced GCE Mathematics											
				Max Mark	**a**	**b**	**c**	**d**	**e**	**u**
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

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AS & Advanced GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0

AS GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

For more information about results and grade calculations, see https://www.ocr.org.uk/students/getting-your-results/

Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866	02	Critical maths	Raw	60	44	39	34	29	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867	02	Statistical problem solving	Raw	60	40	36	32	28	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0